

## Overview of Noise Calculations

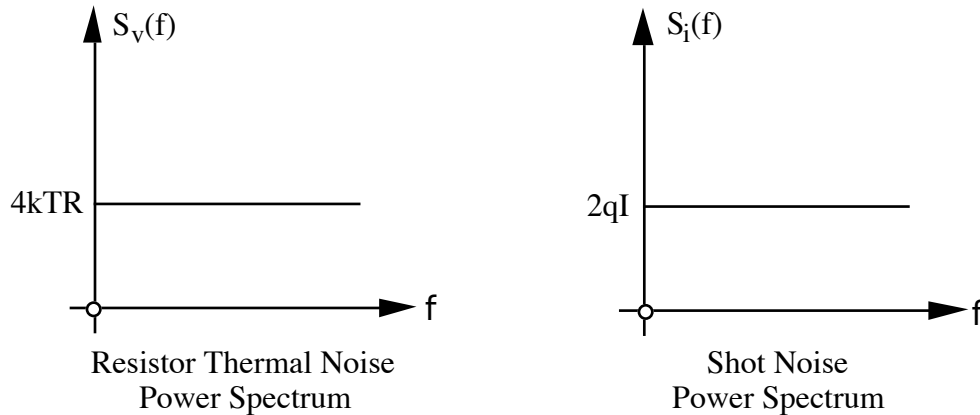
### Parseval's Theorem<sup>1</sup>:

The mean square value of a random variable  $v(t)$  or  $i(t)$  is related to its (one-sided) Power Spectral Density  $S_v(f)$  or  $S_i(f)$  by

$$\overline{v(t)^2} = \int_0^{\infty} S_v(f) df \quad \text{and} \quad \overline{i(t)^2} = \int_0^{\infty} S_i(f) df$$

### White Noise

For "white noise" sources including the thermal noise of resistors or shot noise associated with pn junctions, the power spectra are independent of frequency:



The noise power in a frequency band between  $f_1$  and  $f_2$   $\Delta f = f_2 - f_1$  is given by

$$\overline{v(t)^2} = \int_{f_1}^{f_2} 4kTR df = 4kTR\Delta f \quad \text{and} \quad \overline{i(t)^2} = \int_{f_1}^{f_2} 2qI df = 2qI\Delta f$$

The bandwidth  $\Delta f$  is also often called B in many publications,  $B = f_2 - f_1$ :

$$\overline{v(t)^2} = \int_{f_1}^{f_2} 4kTR df = 4kTRB \quad \text{and} \quad \overline{i(t)^2} = \int_{f_1}^{f_2} 2qI df = 2qIB$$

### Addition of Noise Sources

For  $v_s(t) = v_1(t) + v_2(t)$ ,

$$\overline{v_s^2(t)} = \overline{[v_1(t) + v_2(t)]^2} = \overline{v_1^2(t)} + \overline{v_2^2(t)} + 2\overline{v_1(t)v_2(t)}$$

$$\overline{v_s^2(t)} = \overline{v_1^2(t)} + \overline{v_2^2(t)} + 2C_{12}\sqrt{\overline{v_1^2(t)}\overline{v_2^2(t)}} \quad \text{with} \quad C_{12} = \frac{\overline{v_1(t)v_2(t)}}{\sqrt{\overline{v_1^2(t)}\overline{v_2^2(t)}}} = \text{correlation coefficient}$$

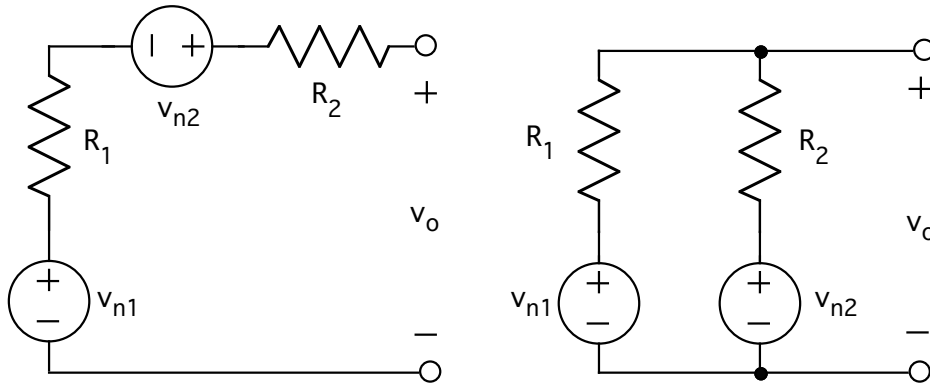
<sup>1</sup> R. D. Thornton, D. Dewitt, E. R. Chenette and P. E. Gray, *Characteristics and Limitations of Transistors*, SEEC Vol. 4, Wiley: 1966.

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$C_{12}$  represents a measure of the correlation between  $v_1$  and  $v_2$ , and  $-1 \leq C_{12} \leq +1$ . For uncorrelated noise sources,  $C_{12} = 0$ , and

For uncorrelated noise sources: 
$$\overline{v_s^2(t)} = \overline{[v_1(t) + v_2(t)]^2} = \overline{v_1^2(t)} + \overline{v_2^2(t)}$$

### Example of Noise Calculations for Resistor Combinations



Series connection:

$$v_o = v_{n1} + v_{n2} \quad \text{where} \quad \overline{v_{n1}^2} = 4kTR_1B \quad \text{and} \quad \overline{v_{n2}^2} = 4kTR_2B$$

Since the resistors are physically independent,

we expect the individual noise sources to be uncorrelated, and

$$\overline{v_o^2} = \overline{(v_{n1} + v_{n2})^2} = \overline{v_{o1}^2} + \overline{v_{o2}^2} = 4kTR_1B + 4kTR_2B = 4kT(R_1 + R_2)B$$

Parallel connection:

$$v_o = v_{n1} \frac{R_2}{R_1 + R_2} + v_{n2} \frac{R_1}{R_1 + R_2}$$

Again, assuming the individual noise sources to be uncorrelated,

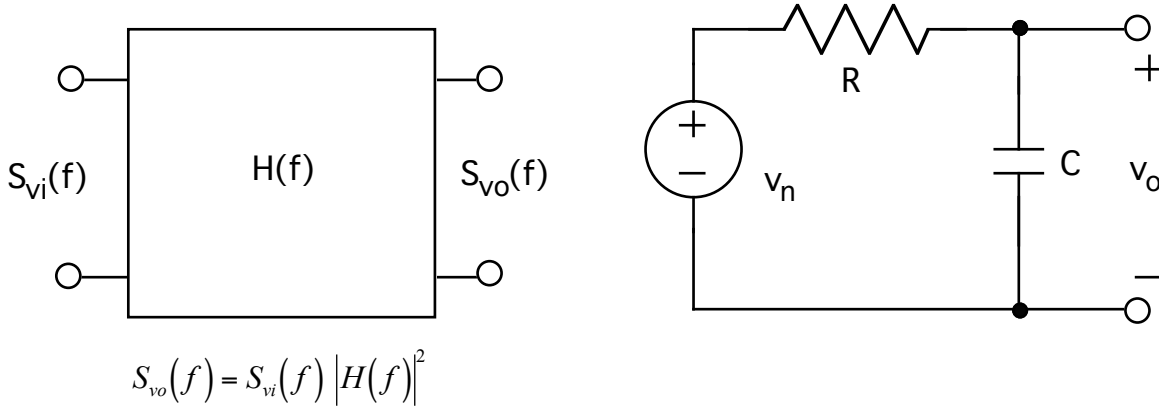
$$\overline{v_o^2} = \overline{\left( v_{n1} \frac{R_2}{R_1 + R_2} + v_{n2} \frac{R_1}{R_1 + R_2} \right)^2} = \overline{v_{o1}^2} \left( \frac{R_2}{R_1 + R_2} \right)^2 + \overline{v_{o2}^2} \left( \frac{R_1}{R_1 + R_2} \right)^2$$

$$\overline{v_o^2} = 4kTB \frac{R_1 R_2^2 + R_1^2 R_2}{(R_1 + R_2)^2} = 4kTB \frac{R_1 R_2}{R_1 + R_2} = 4kT(R_1 || R_2)B$$

## Overview of Noise Calculations

### Noise Calculations in Frequency Dependent Networks

For a linear frequency dependent network, the output power spectral density  $S_{v_o}(f)$  is equal to the input power spectrum  $S_{v_i}(f)$  multiplied by the square of the magnitude of the network transfer function  $H(f)$ :



Consider the low-pass filter in the figure above

$$S_{v_i} = 4kTR \quad H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{1 + sRC} \quad |H(f)|^2 = \frac{1}{1 + (2\pi RCf)^2}$$

$$S_{v_o}(f) = \frac{4kTR}{1 + (2\pi RCf)^2} \quad \overline{v_o^2} = \int_0^{\infty} \frac{4kTR}{1 + (2\pi RCf)^2} df = \frac{kT}{C}$$

### Effective Noise Bandwidth B

For a given noise power, the effective noise bandwidth is defined as  $ENB = \frac{\overline{v_o^2}}{4kTR}$

For the single - pole low - pass filter above with cutoff frequency  $\omega_o = \frac{1}{RC}$ ,

$$ENB = \frac{kT/C}{4kTR} = \frac{1}{4RC} = \frac{\pi}{2} f_o$$

### Available Noise Power

The maximum power available from a resistor is  $\overline{v_n^2} = kTB$

In a 1 - Hz bandwidth at room temperature, the available power corresponds to

$$\overline{v_n^2} = (1.38 \times 10^{-23} \text{ J / K})(300 \text{ K})(1 \text{ Hz}) = 4.14 \times 10^{-21} \text{ W}$$

This value is often expressed in dBm as:  $10 \log \left[ \frac{4.14 \times 10^{-21} \text{ W}}{0.001 \text{ W}} \right] = -174 \text{ dBm}$